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Einsteinian Physics in the Classroom: Integrating Physical and Digital Learning Resources in the Context of an International Research Collaboration

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This study reports on a pilot program conducted by members of the international Einsteinian Physics Education Research (EPER) Collaboration that aims to pool and combine innovative learning approaches in Einsteinian Physics. The collaboration also aims to disseminate learning resources and research results across a range of countries. In this study, we describe an integrated pilot programme that combines physical models and digital resources to explore secondary school students' (Grade 10, 15 years old) conceptual understanding in the learning domain of Einsteinian physics. After the teaching units "gravity and warped time", "gravity is geometry", and "quantum weirdness", we found that students had gained knowledge of key concepts in the learning domain of Einsteinian Physics. The units rely on physical models or digital learning resources. Both approaches proved successful in introducing Einsteinian concepts. By reporting on this integrated programme, we wish to share our model of an international physics education collaboration. Raising awareness for the need and possibility of introducing Einsteinian physics to school curricula, we hope to offer valuable impetus to the field of physics education that will inspire researchers and teachers alike.

Keywords:

1. Introduction

In this section, we present the rationale of the Einsteinian Physics Education Research (EPER) Collaboration and introduce the educational approaches of the three groups in this collaboration that jointly conducted the present study.

1.1. *Einsteinian Physics Education Research (EPER) Collaboration*

Einsteinian physics (EP) comprises our current best understanding of the physical universe. The theories of relativity and quantum physics, pioneered by Albert Einstein, Max Planck and other

scientists,[1,54] stand relevant in today's society because of their importance in scientific and technological advancements. However, physics in schools worldwide remains dominated by the classical Newtonian standpoint, with EP only explored superficially.

In response to the need of finding sound educational approaches that allow middle and secondary school students to learn about EP in schools, physicists and educators from around the world have formed the EPER Collaboration. Members of the EPER collaboration acknowledge the need to find innovative ways to conduct EP education research on a global scale.[2,3] Specifically, the aims of the EPER collaboration are as follows:

(1) To share existing EP learning resources,
(2) To develop novel learning resources for all levels of education,
(3) To develop resources for teachers,
(4) To investigate students' conceptual understanding of EP, and
(5) To disseminate best-practice examples and research results by building an international research network.

The participating research groups have pioneered different approaches for researching, teaching, and learning in EP. In this study, we present an integrated teaching approach from three research groups in Australia, Norway, and Germany.[4–6]

The Einstein-First Project: A group of physicists and physics educators in Australia formed the Einstein-First project to introduce modern concepts of physics by reforming the school science curriculum. This project has developed hands-on learning resources for introducing general relativity and quantum physics in schools.[4,7] Students from Grades 7–12 (12 to 17 years old) have been tested and results show that it is possible to teach EP at this level.[8,9] The project has also performed surveys on public attitudes to implementing EP in schools and teacher training for teaching EP.[10]

Project ReleQuant: ReleQuant is an educational project that has developed digital learning environments about quantum physics and general relativity for upper secondary school students in Norway.[5,11] Methods of design-based research[12] have guided the development of the learning resources that aim to promote qualitative and conceptual understanding in EP.[13,14] ReleQuant employs a sociocultural view towards learning and relies on features of history and philosophy of science to move beyond traditional instructional approaches.[15–18] The Norwegian Centre for Science Education hosts the digital learning environments on the open learning platform www.viten.no.

The Spacetime-Travel Project: The physics education group at Hildesheim University, Germany, runs the project Spacetime-Travel to facilitate the teaching of special and general relativity at school and at university. The group develops novel teaching materials, in particular, visualizations and models; the group also develops teaching modules for school and university education.[19–22] After a process of testing and refinement,[23,24] the resulting media and teaching modules are made available as open educational resources.[25]

2. Literature Review: Conceptual Understanding of EP in the Context of Time, Space, and Quantum Weirdness

In this section, we perform a literature review of students' understanding of key concepts of EP. We specifically discuss the key concepts of EP pertaining to the three units used in this integrated programme.

2.1. *Students' understanding of time in relativity*

Physicists and physics educators have identified the concept of time as crucial in the theory of relativity and particularly difficult for students to grasp.[26–29] A review of the literature on student understanding of time reveals that researchers have mostly looked at conceptions of time in Galilean relativity and special relativity. The construct of a reference frame lies at the heart of relative motion.[30] Thus, the focus of research has been on student understanding of reference frames, simultaneity, and time dilation.

Studies agree that students at all academic levels from middle and secondary school to undergraduate, graduate and pre-service teacher education struggle with the relativity of simultaneity and with the role of observers in inertial reference frames.[26,31–34] Interestingly, the difficulties persist even when students learn about the relativity of time in VR-environments, which offer a completely new way of experiencing physical phenomena.[32] Difficulties often start with the definition of time as

the time of an event or the notion of proper time as the time an observer would read on his or her wristwatch.[30,33] Ideas of absolute simultaneity and the relativity of simultaneity harmoniously co exist.[30] Often and quite spontaneously, students use classical lines of reasoning to explain relativistic kinematics even after the instruction of relativity and ideas of absolute motion are deeply rooted among students.[26,32] Seeing that Hewson already observed in the 80s that student ideas seem to be the "product of Einsteinian branches grafted to Newtonian roots" it is somewhat depressing that physics educators apparently have not succeeded in overcoming these obstacles.[34]

While educators agree that students face serious difficulties in their conceptualization of reference frames and the relativity of time and simultaneity, little research has investigated student understanding of time in general relativity. Suggesting a novel teaching approach to the role of time in GR, Gould claims that many students have difficulties understanding the scale of time.[27] Even though he does not give empirical evidence or any reference for his claims, it is reasonable to assume that students struggle with the idea that time slows down near massive objects. The Australian project Einstein-First has gathered the first evidence that secondary school students have problems to explain why time at the top of a mountain is different compared with its base. In a similar vein, secondary students in Norway found it difficult to conceptualize motion along the time dimension.[14,29] In summary, despite time playing a crucial and conceptually difficult role in the domain of EP, insufficient research has been conducted to understand how students make meaning of time.

2.2. *Students' understanding of non-Euclidean geometry and curvature*

The concept of a curved surface is part of everyday experience and preschool children generally know what is meant by a surface being curved or being plane.[35] Post-instruction tests show that undergraduate students can confidently determine the curvature of surfaces in a qualitative way (i.e. discriminate between positive, negative, and null curvature).[24]

In a study of how students learn concepts of non-Euclidean geometry, Junius[36] concludes, "as Poincaré[37] suggested, one does not learn a new geometry, rather, one gets used to it". The study focuses on the concept of the straight line in spherical geometry and finds that university students' personal experience, here involving physical motion, was instrumental for the development of an understanding of spherical geometry.

Bandyopadhyay and Kumar provided undergraduate students with an illustration of a non-Euclidean continuum and interviewed them to test their understanding.[38] They found amongst other things that students may "doubt the ability to measure distances on a non-Euclidean continuum". Also, students more readily adopt the extrinsic view of an "outside" observer considering the surface as embedded in Euclidean space than the intrinsic view of an observer "in" the surface.

Coble *et al.* have investigated undergraduate students' ideas about the curvature of three-dimensional space in the context of cosmology.[39] In a pre-instruction survey, they found amongst other things that "students' ideas on the meaning of the term curvature were similar to ... its cosmological usage" and that "students are sceptical that the curvature can be measured." In a multiple-choice post-instruction exam question, a majority of students described the Universe as having zero overall curvature.

2.3. *Students' understanding of quantum weirdness*

The way things behave at the quantum level does not match with our everyday experience. As a result, there are numerous misunderstandings induced by the classical approach to interpreting the world. The traditional way of teaching quantum mechanics might also lead to many misconceptions in students' understanding.[40]

A majority of secondary school students (16–18 years old) cannot make a distinction between classical and quantum objects and consider electrons to behave as point-like objects.[41] Bungum *et al.* found that although upper secondary school students have a basic understanding of the central concepts of quantum physics, they lack a qualitative understanding of these concepts.[18] For example, students think that quantum objects travel in a wave-like trajectory. They often try to fit in their concepts with classical reality, an observation made by Mannila *et al.*[42]

These misconceptions might be avoided by adopting modern techniques of instruction. For

instance, through a virtual laboratory instructional method, Muller and Wiesner found that students could acquire appropriate quantum mechanical conceptions.[40] Students were able to understand that quantum mechanics deviates from their usual interpretation of the everyday world.

3. Concepts of the EP Programme

In this section, we discuss the important concepts of Einsteinian physics in the context of the three units of this programme.

3.1. *ReleQuant*: *Gravity and warped time*

Physics is the science of change in time. Yet, time turns out to be a particularly elusive concept in EP. In his special theory of relativity, Einstein posited that time is not absolute. Depending on the movement of observers, time can go slower or faster leading to the phenomenon of time dilation. The concept of simultaneity is not absolute either, and different observers will not be able to agree on whether events happen at the same time. In his general theory of relativity, Einstein observed that time and space are not static entities against which the laws of physics unfold. Rather, time and space are dynamic and respond to the presence of massive objects. Massive objects distort the fabric of spacetime and it is this distortion, which leads to gravitational phenomena. Both time and space can be curved and stretched and the gravitational field determines the way time flows.

The link between gravity and warped time is interesting to explore further. Many objects in the universe, such as the Earth, are not massive enough to curve space considerably. We experience gravity mainly because of warped time that causes us to accelerate downwards. Researchers from ReleQuant have developed an interactive and digital model that illustrates warped time and its relation to gravity.[5]

The warped-time model is part of a digital learning environment in general relativity that is accessible at www.viten.no/relativity. The warped-time model invites students to explore the physics of free fall both from a classical and from a relativistic perspective by warping the time axis of a digital height-time diagram.[5] The model does not attempt to be rigorous but offers a qualitative approach to convey the key idea that freely falling objects follow geodesic curves through spacetime.

3.2. *Spacetime travel: Gravity is geometry*

General relativity is Einstein's theory of gravity that, in contrast to the classical Newtonian theory, describes gravity in terms of the geometry of spacetime, not in terms of a force. Wheeler in 1990,[43] gave a concise description of general relativity, stating, "*Spacetime tells matter how to move*, *matter tells spacetime how to curve*". This statement expresses two key ideas that are addressed in the integrated programme on which we report. The first key idea, "*Spacetime tells matter how to move*" is treated in the unit "Time and Gravity" from the ReleQuant programme (see above).

The unit "Gravity is Geometry" from the Spacetime-Travel relativity programme introduces the second key idea: "*Matter tells spacetime how to curve*". Here "spacetime curves" is short for a spacetime that does not have the Minkowskian geometry described in special relativity. The difference from Minkowskian geometry is quantified by the curvature of the spacetime. The curvature, in turn, is related to the matter content. Einstein's field equations express the connection between curvature and matter. They are at the core of general relativity.

This unit aims to introduce the concept of curvature and to show how matter and curvature are related according to the field equations. We use sector models to visualize curved spaces, and a local formulation of the field equations, focusing on one equation (out of the set of ten). This unit is a workshop developed by the Spacetime-Travel Project and described in Refs. 6 and 45.

3.3. *Einstein-First*: *Quantum weirdness*

Modern experiments have shown that single photons (light quanta) arrive randomly and create interference patterns.[55] Not only photons but also every object in the universe can create interference. Particles as massive as phthalocyanine molecules have shown to create interference patterns.[46] The emergence of interference (a wave-like property) can be understood without losing the particle characteristics of radiation. The relationship between wave-

like and particle-like characteristics is expressed by de Broglie's universal relationship

$$\text{Wavelength} = \text{Planck's constant/Momentum.}$$
$$\text{or} \quad \lambda = \frac{h}{p}.$$

In contrast to classical objects for which we can determine their exact position provided we know their initial conditions, we can only determine the *probability* of finding a quantum object at a certain point.

In our program, we have chosen to use Feynman's path integral formulation of quantum mechanics because it provides a rule-based procedure for directly calculating the quantum probability amplitude of quanta (a particle or a photon) appearing at a certain location. The method can be directly applied to modern observations such as single-photon interference, and requires only one correspondence to be made: that between classical wave amplitudes and quantum probability amplitudes.

The path integral approach is directly analogous to the summation of multiple classical waves at a point and uses the method of *phasors* independently proposed by Heaviside and Steinmetz in 1893.[47,48] Phasors themselves can be made intuitive through the use of simple physical phasor wheels in which rotating phasor vectors are directly linked to simple harmonic motion. The vector summation of classical waves then translates directly to the quantum probability of quanta arriving at a point. As emphasised by Feynman in his book QED: Strange Theory of Light and Matter,[49] students are asked to accept just one rule: that the probability of a particle appearing at a particular point in space is determined by a vector sum of phasors for all possible paths to that point. A detailed paper describing the path integral approach to teaching quantum physics in high school will be published in Physics Education.

4. Motivation and Research Objectives

Here we discuss the motivation for this study before we provide the research questions that guided our study.

4.1. *The motivation for this study*

Independent trials on introducing EP based on the aforementioned resources have not been performed at Grade 10 level. Our collaborative effort has the potential to provide valuable information for introducing EP at the lower secondary level. First, by combining three different instructional resources and conducting educational research at the lower secondary level, we aim to fill that gap in the educational landscape. Second, we wish to provide support for teachers and practitioners to introduce topics of Einsteinian physics to their classrooms. Third, our collaborative effort has the potential to provide best-practice examples, which can act as a strong platform for building an international network of EP education.

4.2. *Research questions*

The overarching aim of this joint research project is to analyze students' first-hand experience of an integrated EP programme. The outcomes provide insight into students' challenges in the learning domain of EP which teachers can build on to improve their instructional practices. Additionally, our findings can inform further refinement of the educational programmes. Such refinements and iterative improvements correspond to the goal of the EPER collaboration to develop successful instructional resources and disseminate these resources.

The overarching aim of this research project served as guidance to formulate one RQ for each teaching unit as follows:

RQ1. How do participating students respond to the digital warped time model as a tool to illustrate warped time?

RQ 2: How successfully do participating students use sector models as a tool to determine the curvature of three-dimensional curved spaces?

RQ 3: How do participating students respond to phasors as a tool to illustrate the quantum probability of photons?

5. Methods

Models, analogies, and digital resources are considered valuable tools for students to approach abstract concepts and to build conceptual understanding.[5] These tools have a prominent place in the units of this programme and are the basis for activity-based learning that we use to invite

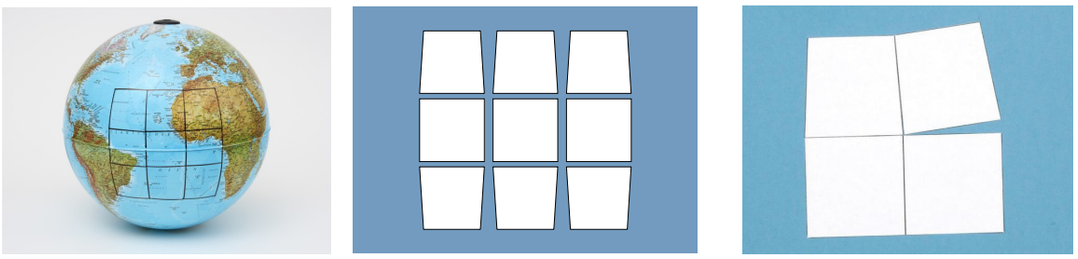

Fig. 1. The determination of curvature using sector models. (Left): The sphere is the prototype of a surface with positive curvature. (Middle): Sector model of a spherical cap. To create the sector model, the sphere is subdivided into small pieces of area, each piece is approximated by a flat piece, and the flat pieces (the sectors) are laid out in the plane. (Right): When the sectors are assembled around a vertex, a gap remains. This indicates positive curvature. In the case of a saddle, the assembled sectors would overlap, indicating negative curvature.

students to gain knowledge and develop understanding of EP concepts.

In this section, we give an overview of the entire programme and present data collection and analysis procedure.

5.1. *Overview of the programme*

The integrated programme was divided into three units of two lessons each followed by a single review lesson. Each lesson lasted for one hour and time was equally divided for each unit. The units are described briefly as follows:

Unit 1: *Time and Gravity.* We used web-based learning resources to teach the concept of time and gravity developed by Project ReleQuant.[5] Students were instructed to complete the 90 minutes module in two lessons. At the end of the module, the students were expected to:

(a) Be able to describe general relativity as a theory where space, time and gravity are interlinked
(b) Be able to describe space as three spatial dimensions and time as the fourth dimension
(c) Be able to explain gravity as a geometric phenomenon.

Students worked in small groups to explore digital content. The module guided the students through a series of activities. The groups of students were free to work through these activities at their own pace; however, they had the opportunity to consult and discuss with the teachers and researchers at any time. The instructional approach to teaching curved space and warped time is described in more detail by Kersting *et al.*[5]

Unit 2: *Gravity is Geometry.* This unit is a workshop developed by the Spacetime-Travel Project and described in more detail by Zahn and Kraus[6] and Kraus and Zahn.[45] In this unit, we addressed a core component of general relativity: Einstein's field equation. Part one of the unit introduced the basic concepts using curved surfaces as examples. A criterion for the qualitative determination of curvature was given and students applied it to different examples, presented as physical models. Then the sector model of a curved surface was introduced and students applied the criterion for curvature to sector models. These models were provided in the form of sectors cut out from paper (see Fig. 1).

The second part of the unit treated curved three-dimensional space. The concept of the sector model was translated from two to three dimensions, i.e. from flat pieces of paper to blocks made of cardboard. The criterion for curvature was applied to the three-dimensional sector model for students to realize that curvature must be tested in three directions: they obtained three curvature components. Lastly, one of the ten field equations (see Ref. 45) was introduced and applied to: the space around a black hole and the interior of a neutron star.

At the end of the lesson, students were expected to:

(a) Be able to describe the curvature of a surface (as positive, negative, or null), when shown a physical model or a sector model of the surface.
(b) Be able to describe the curvature of a three-dimensional space via its three spatial components, when given the sector model of the space.
(c) Be able to use Einstein's field equation to deduce if matter is present in space when given the sector model of the space.

Unit 3: *Quantum Weirdness.* The main components of this unit were to emphasize (a) de Broglie's

Fig. 2. (Right) Phasor-wheel model. A circular wheel with a handle and an arrow marked on it. A linear stick attached to the wheel makes a sinusoidal wave-like pattern when projected on a wall as it moves forward. (Left). A cup-lid wheel, same as the wooden wheel, can be used to trace different path lengths on an A3 paper.

hypothesis, and (b) the probability of photon arrival at a particular location can be described by the mathematics of waves, using phasors.

In the first part, we showed videos of water-wave interference, single-photon interference (www.youtube.com/watch?v = MbLzh1Y9POQ) and particle (phthalocyanine) interference (www.youtube.com/watch?v = NUS6_S1KzC8). Then, we used physical models in the form of phasor wheels, to describe phasors.[50,51] A cup-lid wheel (see Fig. 2) carries a phasor arrow on a radius line, a ruler for support and a vertical drinking straw to trace the projected sinusoidal motion.

Students observed the connection between wave motion and a rotating wheel. They used the phasor wheels to trace the paths of two waves to a point. They graphically summed up the phasors to study the addition or cancellation of the waves. They explored that the addition of phasors was equivalent to the adding waves.

The phasor activity is described in this video available at https://youtu.be/AVmaV_JS7Cg.[51] Students rotate the wheels to trace pairs of trajectories corresponding to a double slit experiment and marked the corresponding phasors. They plotted the two phasors and determined the resultant phasor.

The classical and quantum interpretations were then contrasted. For waves like water-waves, the resultant is proportional to the wave amplitude, as is easily seen in two-source water-wave interference videos. For quantum particles, the same result represents the probability of a photon appearing at a particular location. Similar patterns can be seen accumulating in the single-photon and molecule interference videos.[46,55] The maths is identical but the interpretation is completely different. We can see why the water waves add as they do, but we cannot understand why the photons or molecules act in this way. This is the quantum weirdness we have to accept as an observational reality.

It is to be noted that to prevent information overload in the short available time, we did not emphasize the final aspect: the square of the resultant is proportional to the wave intensity (classical waves), or the probability, or the number of quanta per second at each location. Teachers that have more time to address these concepts could supplement the programme with this final aspect.

Worksheets were used to explain the rules of phasors and photon trajectories in the lesson.

At the end of the lesson, students were expected to:

(a) Be able to describe the de Broglie relationship: everything has wave and particle nature
(b) Be able to describe phasors and how they can be used to add waves.
(c) Be able to use phasors to add waves as well as to find probability amplitude of arriving photons

5.2. *Data collection*

We used questionnaires after each of the three units to collect written responses from the students. For the unit "gravity and warped time" open-ended

questions were used that invited students to elaborate on the concepts in question, thus probing their conceptual understanding.[52] For the unit "gravity is geometry", closed questions were used to test the students' understanding of the explanations they had been given by requiring them to use the explanations with new examples. For the unit "quantum weirdness", open-ended questions were used to test student understanding by requiring students to describe and explain key aspects of the activity that they had performed.

We also used survey questionnaires for teachers to collect their opinion.

Two secondary schools in Perth, Western Australia were chosen with 72 students, 50 and 22 from each school. In the first school, the group of 50 students was divided into smaller groups of 16 or 17 students for every lesson, while in the second school all the 22 students were instructed together in one group.

Students' consent was obtained for research through human research ethics approval issued by the University of Western Australia.

5.3. *Data analysis*

Responses to the closed questions were analyzed based on a point-scheme. One point was assigned for every correct response and additional one point for providing an explanation. For example when asked, "Does this space contain matter? Give a reason for your answer" the respondent was assigned 2-points if both parts were correct, 1-point if just one part, and zero if none.

The open-ended questions were interpreted using thematic analysis, as described by Braun and Clarke.[53] The research questions guided the analysis of written responses for each question. Individual themes were identified and coded to find patterned responses within the data sets. In this way, we obtained the key features of student understanding which we unpack in the next section.

6. Results and Discussion

In this section, we present the findings of our joint programme and discuss implications for instructional practices.

6.1. *Time and gravity*

We asked the following four questions to analyze student understanding of time and gravity in the first unit.

Q1 What is time?
Q2 What is gravity?
Q3 How does gravity make things fall?
Q4 Are you moving in spacetime right now?

Because of the open nature of the questions, student answers varied to some degree as can be expected in a learning domain that still poses many open questions to physicists. The thematic analysis allowed us to classify the responses in line with recurring key concepts. We identified three key concepts out of which the first two were based on Q1 and Q2, and the third was based on Q3 and Q4. They are discussed as follows:

Time as the fourth dimension: As mentioned in the literature review section, students often struggle to understand the concept of time. Students provided a range of responses for Q1 such as the following.

"*Time is a singular dimension which works in conjunction with three dimensions of space*"

"*Time is related to space and changes under different gravitational pull*"

"*Time is a concept which is used to describe how long an object takes to get from one place to another*"

"*Time is a progression of events*"

Students' responses ranged from time as a scalar quantity to time as the fourth dimension. 53% of the total students wrote that time is the fourth dimension or a relative concept or both, whereas 15% answered that time is a scalar quantity and constantly moving forward. Students used various ways of expressing the dependence of time on gravitational potential, including that it "depends on matter", that it changes under different "gravitational pull". All these answers are in line with the Einsteinian concept.

A few others gave diverse and thoughtful responses that were not incorrect but did not explicitly cover the concepts of dimension and relativity. 5% of the total students did not provide an explanation in line with the Einsteinian physics. For example, two students wrote "not sure" and one students wrote, "Time is what we see on the watch".

Gravity as warping of spacetime: Students were able to formulate an Einsteinian model of gravity. Many students wrote that gravity arises due to

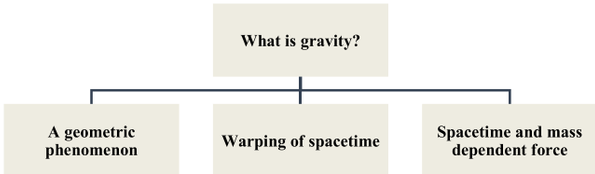

Fig. 3. Students explained the concept of gravity in an Einsteinian context. Other responses explained gravity as (a) a geometric phenomenon, (b) warping of spacetime, (c) spacetime dependent force. These responses sum up their approach to describing gravity, which is in line with the Einsteinian model of gravity.

the curvature of spacetime. 90% of the total students wrote that gravity is a spacetime dependent force or dependent on matter or a geometric phenomenon. We identified three recurring responses in their explanation shown in Fig. 3.

The remaining 10% of the students' answers were not in line with the Einsteinian description. Overall, their responses indicate that students described gravity as the interplay between matter, space and time.

Objects follow geodesics in spacetime: For Q3 and Q4, 18% of the total students were explicit about objects following a geodesic or a straight line in spacetime. Instead of writing geodesics in spacetime, they wrote that objects fall because of the curvature of spacetime.

When asked, "Are we moving in spacetime right now?" 91% of the students provided correct responses; however, only 14% gave a reason for their answer. Q4 emphasizes that this was a poorly framed question because it did not request an explanation.

Overall, the two lessons were only sufficient to develop a basic understanding of time and gravity. This implies students require more time to achieve a deeper understanding of Einsteinian physics. For instance, they were able to achieve an understanding of spacetime curvature but this was not deep enough for an understanding of geodesics in spacetime.

6.2. *Gravity is geometry*

We used two sets of questions to test student understanding of curvature of space and of the field equations. Both were closed questions based on tasks that required students to apply their knowledge about curvature (taught in the unit) and their skills to use sector models to a new example.

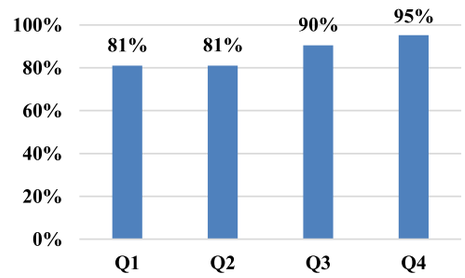

Fig. 4. Percentage of students with correct responses for each question. Results indicate that students were able to use sector models to determine curvature and that they were able to relate curvature to the presence of matter.

For the first set of questions, students were provided with the sector model of a curved surface (representing part of a torus, but they were not given this information) and were asked the following:

(Q1) Describe the curvature of this surface — is it positive, negative, or null?
(Q2) Give a reason for your answer. Mark the sectors that you consider for your answer.

For the second set of questions, students were provided with the sector model of a curved three-dimensional space (representing space in the interior of a neutron star, again, this information was not given to the students) and were asked.

(Q3) What is the curvature of this space? Give a reason for your answer. You can also use a sketch to illustrate what you did.
(Q4) Does this space contain matter? Give a reason for your answer.

We observed that 80% of the students were able to determine the curvature of the surface (Q1, Q2). 90% were able to find the curvature of the space with the sector models provided (Q3). 95% of the students were able to conclude from the field equation that the curvature of space represented by the given sector models implies the presence of matter (Q4).

Students were able to use the models to learn about the curvature of surfaces. They could master the steps from curved surfaces to curved spaces. Finally, the students demonstrated working knowledge of the introduced field equation in the lesson by successfully applying it to an unfamiliar situation. This trial suggests that sector models are practical and useful tools for teaching curved geometry and space even for Grade 10 students.

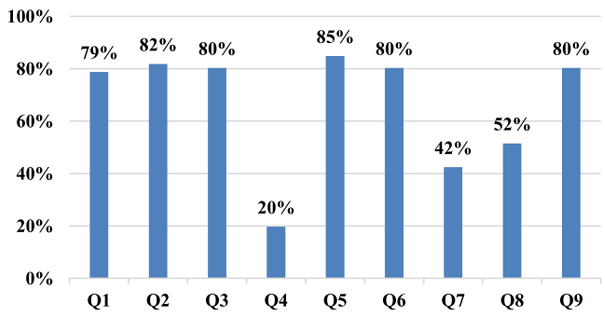

Fig. 5. Percentage of students who were able to answer each question. Q1, Q2, Q3, Q5, and Q6 show that they were able to learn about phasors and their rules of adding or canceling. Q4 shows that two lessons were insufficient for teaching how the resultant of phasors represents the probability of photons. Similarly, in Q7 and Q8 the students performed comparatively low. Lastly, Q9 implies that 80% of the students agree that phasor-wheel activity is useful for teaching quantum weirdness.

6.3. *Quantum weirdness*

In this unit, we used eight questions (see below) to test students' understanding. In addition, we used question no. 9 for their feedback on the phasor-wheel activity.

(Q1) Describe the connection between a rotating wheel, a wave, and a phasor.
(Q2) What happens when two waves of the same amplitude come together so that one wave is at its peak and the other is at its trough? Explain in a few words.
(Q3) Draw two arrows that describe the situation.
(Q4) What does the length of the resultant of two arrows combining mean?
(Q5) Draw four phasors showing how four equal-sized waves combine to make the biggest possible wave.
(Q6) Now draw the phasors showing how the waves could add up to zero.
(Q7) Explain why it seems weird that the patterns you get when water waves combine are similar to the patterns you get when single photons or molecules can take two or more alternative paths.
(Q8) Explain the evidence that things like atoms act like both waves and particles.
(Q9) Was the wheel activity useful to understand about waves?

79% of the total participants were able to explain about waves and phasors introduced in the programme. A few common responses were:

"*The wheel changes the angle of the phasors that determines the path of the waves*"

"*As the wheel spins and moves forward, it creates the waves and shows the phases*"

"*The angle of the rotating wheel corresponds to the phasors*"

"*A phasor is like a vector which can change direction when rotated around a wheel and therefore cause a wave to occur*"

The histogram (Q1) shows students' performance and their ability to draw the connection between waves, phasors, and phasor-wheel activity.

We also tested students' analytical ability to add two or more waves using the concept of phasors. The histograms representing (Q2) and (Q3) indicate that more than 80% of the students could find the resultant of two waves adding or canceling each other, as well as draw corresponding phasors for different points in the waves.

Similarly, (Q5) and (Q6) were based on students' ability to work with phasors. More than 80% of the students could add and cancel phasors to find correct resultants.

An important quantum concept is that the square of the resultant is proportional to the probability of photons arriving at a certain location in space. However, we gave very little time to this concept mainly due to lack of time and information overload. As a result, only 20% of the entire cohort (see Q4) were explicit about this concept. It is also likely that the question was not well framed and the students did not understand it properly.

42% of the students (Q7) could explain that interference emerging from water waves and particles are similar but weird because of the contrasting nature of interfering entities (i.e. waves and particles). However, 52% of the total students (Q8) could explain about an atom's wave-like and particle-like characteristics.

Results indicate that a two-lesson unit was sufficient to address the concept of phasors. However, two lessons were insufficient to address the concepts of de Broglie's relation and quantum probability. This observation is similar to the findings of unit 1 where students were able to describe gravity and time, but were inexplicit in explaining how objects follow geodesics in curved spacetime.

The response to the survey question (Q9) shows that 80% of the students found the phasor-wheel

activity to be useful for learning about quantum path integrals.

6.4. *Teachers' opinion about the program*

We supplied survey questionnaires to teachers' to obtain their feedback and improve the program for the future. Two out of four teachers filled in the questionnaires. The teachers expressed that the digital interactive resources had a solid built-in pedagogy and appreciated the ease with which the students could use the hands-on resources. They requested additional materials in the form of hard copies and workshops for teacher training before they taught the concepts. One of them suggested to test whether students could apply the understanding of the key points into a new context, to achieve a deeper understanding.

7. Conclusion

In this paper, we have reported on a successful implementation of a specialised and diverse educational programme for teaching Einsteinian physics developed by the EPER collaboration. The first four lessons covered warped time and curved spaces in general relativity and had a solid built-in pedagogy, which yielded positive results. The last two lessons introduced the method of quantum path integrals for quantum interference. It was our first trial for teaching quantum mechanics using a phasor-wheel for the path integral approach.

The ReleQuant interactive digital learning resources were successful in introducing the concepts of time and gravity to Grade 10 students in line with an Einsteinian perspective and in line with the formulated learning goals. We found that two lessons were not sufficient to achieve a deeper understanding of time and gravity. The Spacetime-Travel workshop based on sector models as visualization of curved spaces demonstrated that students were quickly able to appreciate the geometry of curved spaces. They were able to apply their understanding to a new situation and relate curvature to the presence of matter. Students were also able to catch on and use the hands-on model of phasors. 80% of them agreed that the phasor-wheel activity is an excellent tool for teaching phasors. The connection between quantum probability and phasors is more sophisticated and clearly two lessons were not adequate to promote a deeper understanding. We used the results of this study to develop an extended six-lesson program on Feynman path integrals, which will be reported in future.

Overall, this study demonstrates the possibility of combining diverse learning resources developed independently around the world. It shows the significance of tactile geometry and digital resources in teaching Einsteinian physics and potential benefits if used in complement to each other. For instance, the introduced resources will facilitate students' understanding of the nature of geometry in curved spaces and how gravity is linked to the warping of the time dimension. Our results also provide further evidence regarding the possibility of teaching Einsteinian physics at lower secondary school level. The two teachers who participated in our survey provided positive assessments based on their observations of the classes. Their advice regarding teacher training and new context testing will be used to further develop this program.

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